

# The Helical Dipole Field

Laplace's Equation (cylindrical coordinates) for the scalar potential:

$$\frac{\partial^2 \Phi}{\partial r^2} + \frac{1}{r} \frac{\partial \Phi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \Phi}{\partial \phi^2} + \frac{\partial^2 \Phi}{\partial z^2} = 0.$$

Separation of variables,  $\Phi = R(r)Q(\phi)Z(z)$ , gives

$$\begin{aligned} \frac{d^2 R}{dr^2} + \frac{1}{r} \frac{dR}{dr} - \left( k^2 + \frac{\nu^2}{r^2} \right) R &= 0, \\ \frac{d^2 Q}{d\phi^2} + \nu^2 Q &= 0, \\ \frac{d^2 Z}{dz^2} + k^2 Z &= 0. \end{aligned}$$

**Note:**

- Repeat period of helical field:  $\lambda = 2\pi/|k|$
- $\nu = 1$  since central dipole field repeats after  $\phi = 2\pi$
- Assume  $B_0$  points vertically upward at  $z = 0$
- $\vec{B} = -\nabla\Phi$  gives...

$$\begin{aligned} B_r &= 2B_0[I_0(kr) - \frac{I_1(kr)}{kr}](\cos kz \sin \phi - \sin kz \cos \phi) \\ B_\phi &= 2B_0 \frac{I_1(kr)}{kr}(\cos kz \cos \phi + \sin kz \sin \phi) \\ B_z &= -2B_0 I_1(kr)(\cos kz \cos \phi + \sin kz \sin \phi) \end{aligned}$$

Look at  $B_x = B_r \cos \phi - B_\phi \sin \phi$ ,  $B_y = B_r \sin \phi + B_\phi \cos \phi$  and expand these fields near the longitudinal axis ( $x, y$  small); then,

$$\begin{aligned} B_x &\approx -B_0 \left\{ [1 + \frac{k^2}{8}(3x^2 + y^2)] \sin kz - \frac{k^2}{4}xy \cos kz \right\} \\ B_y &\approx B_0 \left\{ [1 + \frac{k^2}{8}(x^2 + 3y^2)] \cos kz - \frac{k^2}{4}xy \sin kz \right\} \\ B_z &\approx -B_0 k \left\{ 1 + \frac{k^2}{8}(x^2 + y^2) \right\} [x \cos kz + y \sin kz] \end{aligned}$$

# Helical Dipole Magnets for Spin Manipulations

## Equations of Motion for Particle Trajectory:

Suppose field is of the form ( $s$  = longitudinal coordinate):

$$B_y = B_0 \cos ks$$

$$B_x = -B_0 \sin ks \quad (\text{vertical at ends of magnet})$$

Then the equations of motion are:

$$\frac{d^2x}{ds^2} = -\frac{B_0}{(B\rho)} \cos ks$$

$$\frac{d^2y}{ds^2} = -\frac{B_0}{(B\rho)} \sin ks$$

which have **solutions**:

$$x(s) = x_0 - \frac{B_0}{(B\rho)} \frac{1}{k^2} (1 - \cos ks) + x'_0 s$$

$$y(s) = y_0 + \frac{B_0}{(B\rho)} \frac{1}{k^2} \sin ks + (y'_0 - \frac{B_0}{(B\rho)} \frac{1}{k}) s$$

The outgoing trajectory after passing through a helical magnet of length  $L = 2\pi/k$  is thus

$$\begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix} = \begin{pmatrix} 1 & L & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -\delta \\ 0 \end{pmatrix} \quad \text{where} \quad \delta = \frac{B_0}{k(B\rho)}$$

=====> through a drift of length  $L$ , then a displacement of amount  $-\delta$

## Transformation of Spin Vector:

if  $\mathbf{S} = \psi^\dagger \boldsymbol{\sigma} \psi$  then the equation of motion for the spinor  $\psi$  through a helical dipole is [Courant]:

$$\frac{d\psi}{ds} = \frac{i}{2} \kappa (\boldsymbol{\sigma}_2 \cos ks + \boldsymbol{\sigma}_3 \sin ks) \psi$$

where 
$$\kappa = \frac{B_0(1+G\gamma)}{(B\rho)} .$$

If we write  $\psi_2 = M \psi_1$ , then, for the helix of length  $L$ ,

$$M = \exp\left[\frac{i}{2}(\kappa\boldsymbol{\sigma}_2 + k\boldsymbol{\sigma}_1)L\right]$$

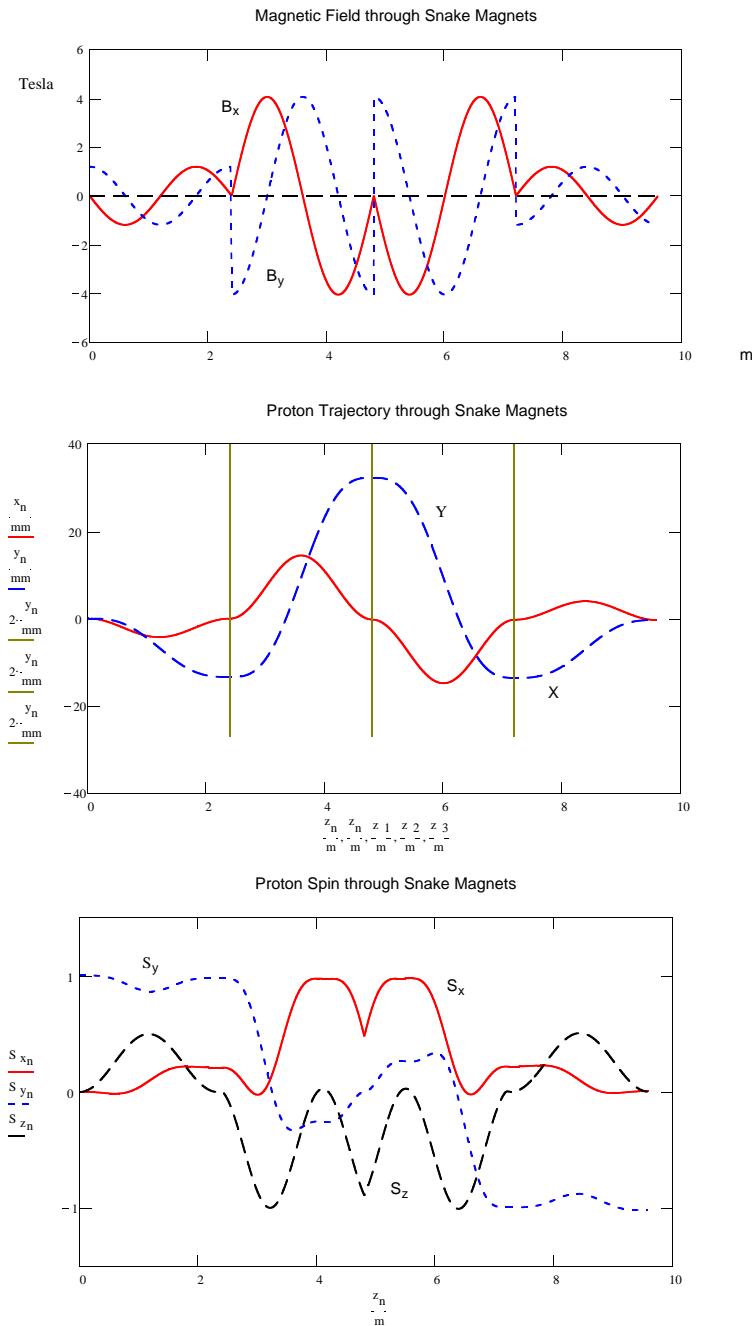
A system of such magnets which has reflection symmetry will always have its rotation axis in the horizontal plane -- the vertical field component is *antisymmetric* about the center, and the horizontal field component is *symmetric*.

Thus, use a system of four helical dipole magnets:

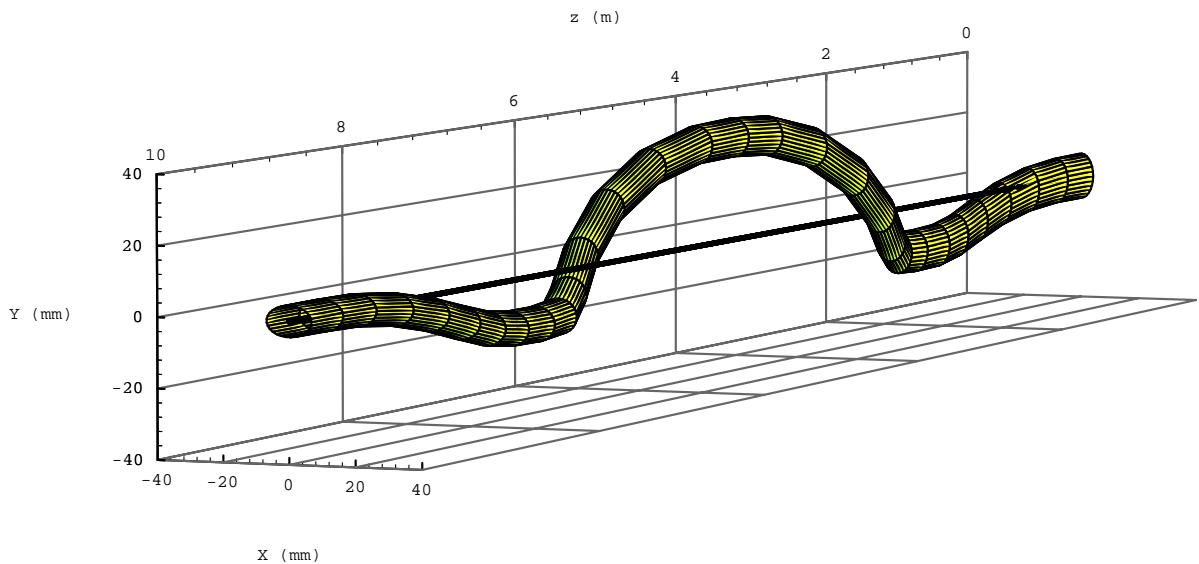
$$B_4 = -B_1, \quad B_3 = -B_2 \quad \implies \text{Total Orbit Distortion} = 0$$

$B_1$  and  $B_2 \quad \implies$  two variables to determine rotation axis and rotation angle of spin vector

## Solutions for Siberian Snakes in RHIC:

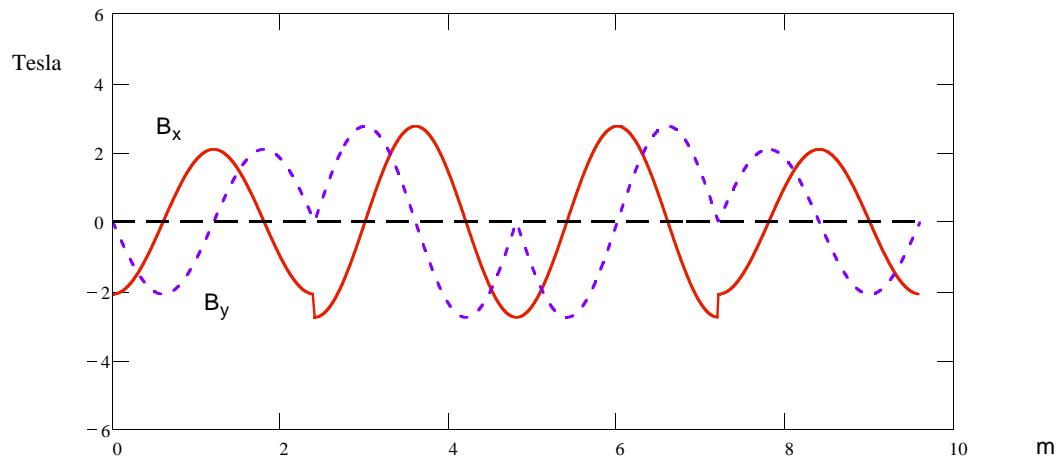


The term “Siberian Snake” was dubbed by Courant in reference to the inventors of the device (Derbenev, et al.) and to the fact that the trajectory “snakes” along the otherwise smooth orbit of the accelerator. Plotting the RHIC Snake trajectory in 3-D illustrates this quite clearly:

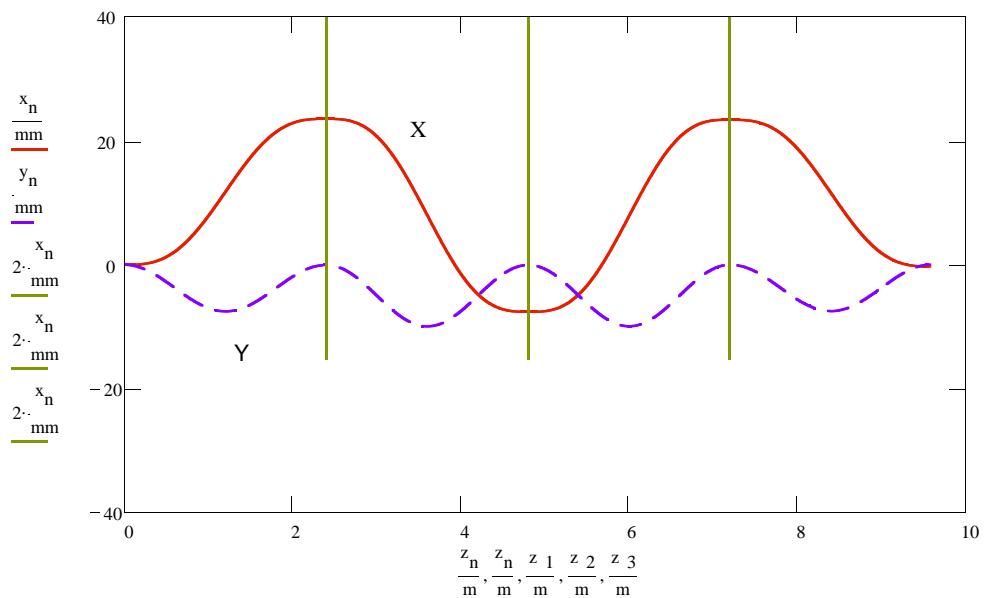


At injection into RHIC, the maximum orbit excursions within the Snake are about 30 mm. The beam cross-section varies along the length due to the transverse focusing structure in RHIC.

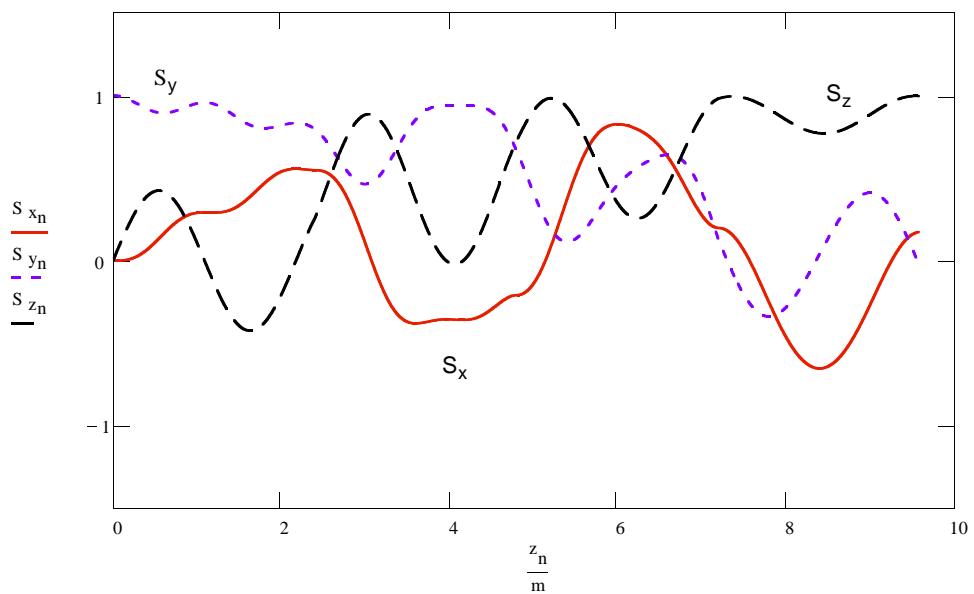
### Magnetic Field through Rotator Magnets

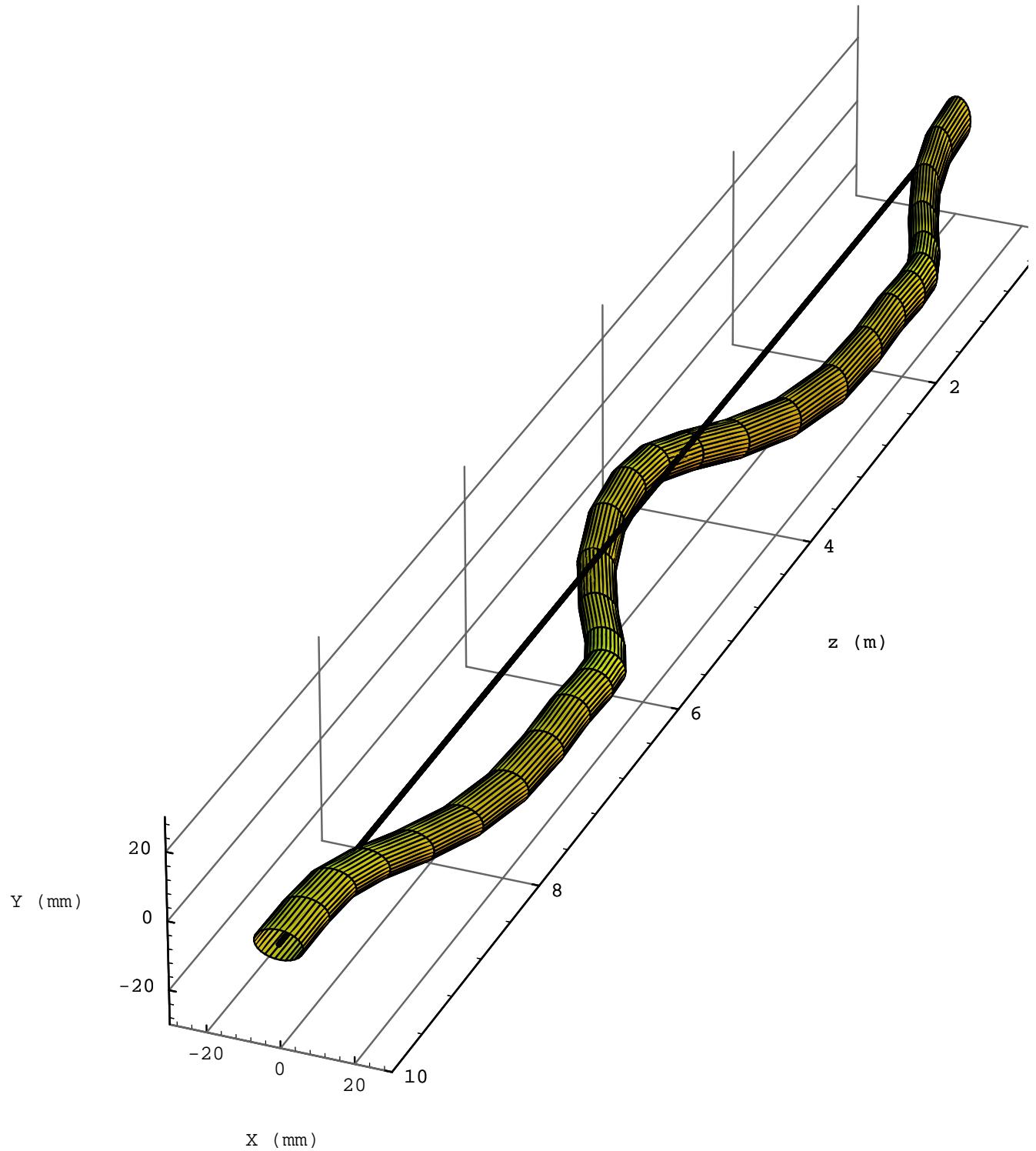


### Proton Trajectory through Rotator Magnets

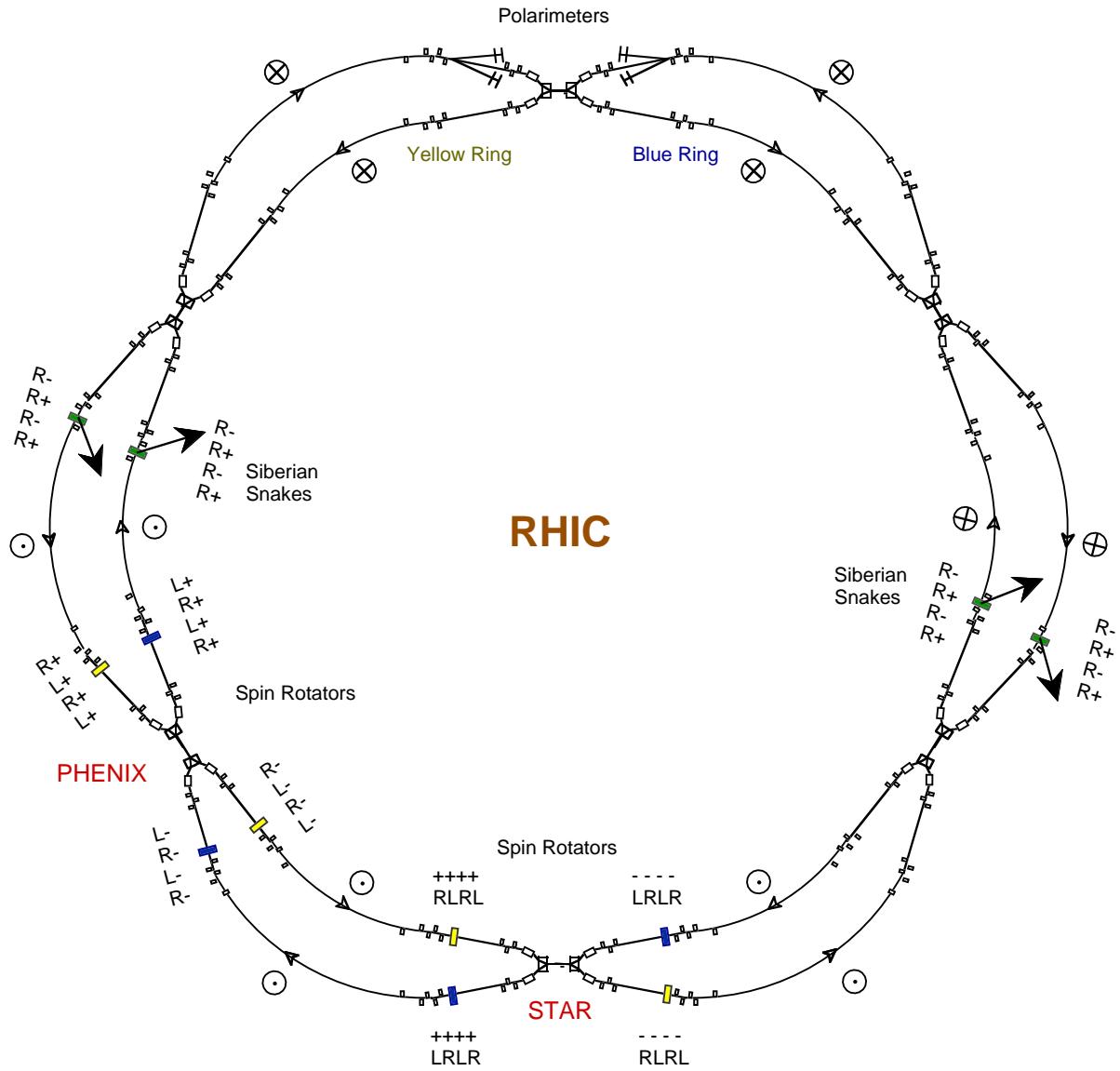


### Proton Spin through Rotator Magnets





A schematic layout of RHIC showing the placement of Snakes as well as Spin Rotators (to make local longitudinal polarization). The designations "L" and "R" denote the "handedness" of the helical dipole magnets.

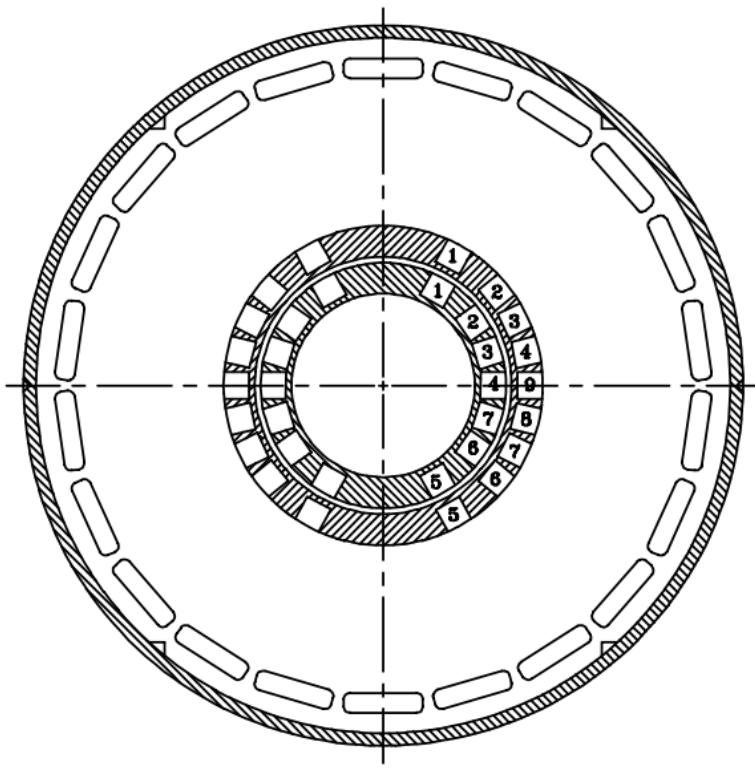


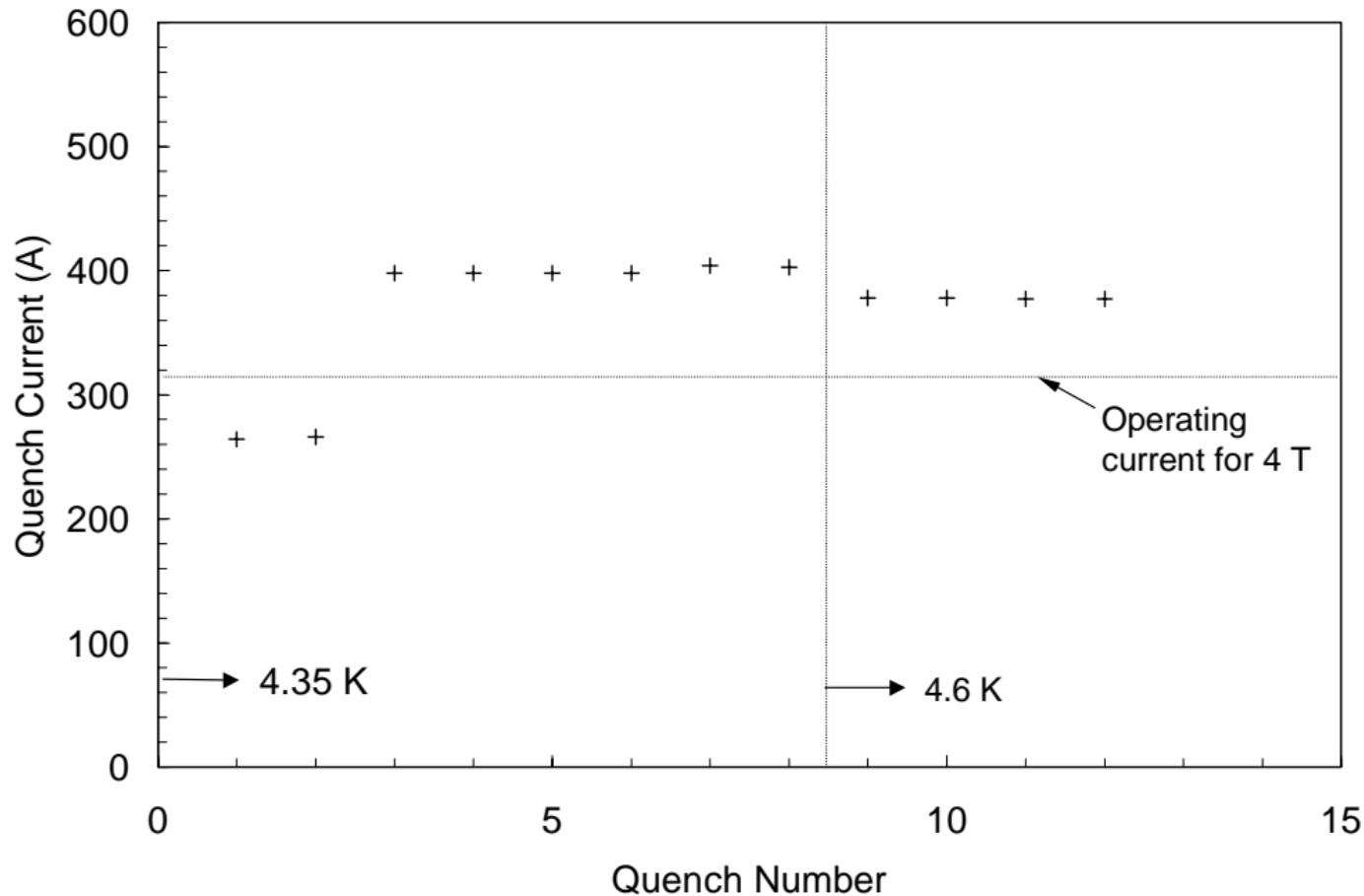
Rotators = Hor field (at ends), + = radially "out," - = radially "in"  
 Snakes = Ver field (at ends), + = "up," - = "down"

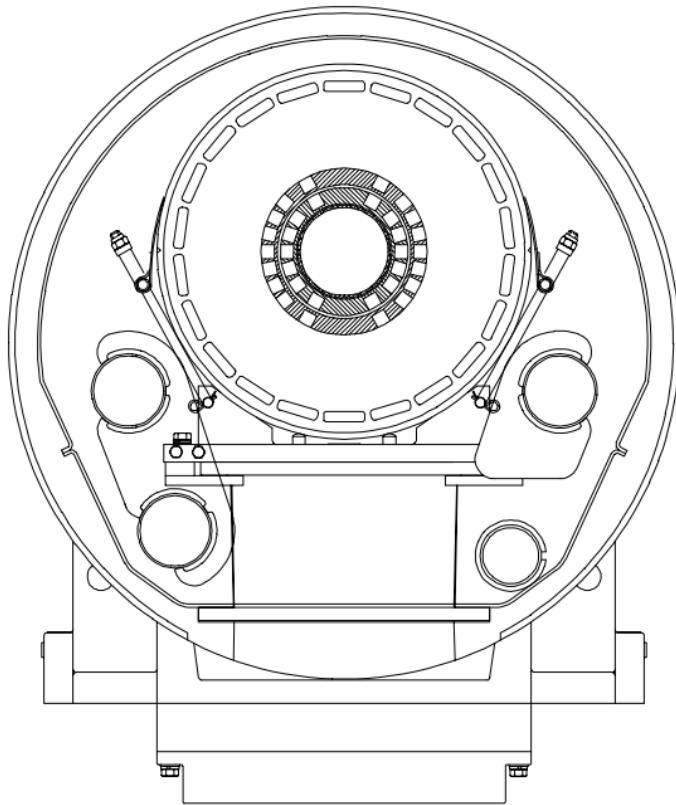
Construction of the helical dipoles is presently taking place in the RHIC Magnet Group at BNL. The winding machine shown below is being used to place superconducting cable into helical grooves machined into aluminum cylinders. Two concentric cylinders surrounded by iron laminations are used to produce the central field of 4 Tesla.



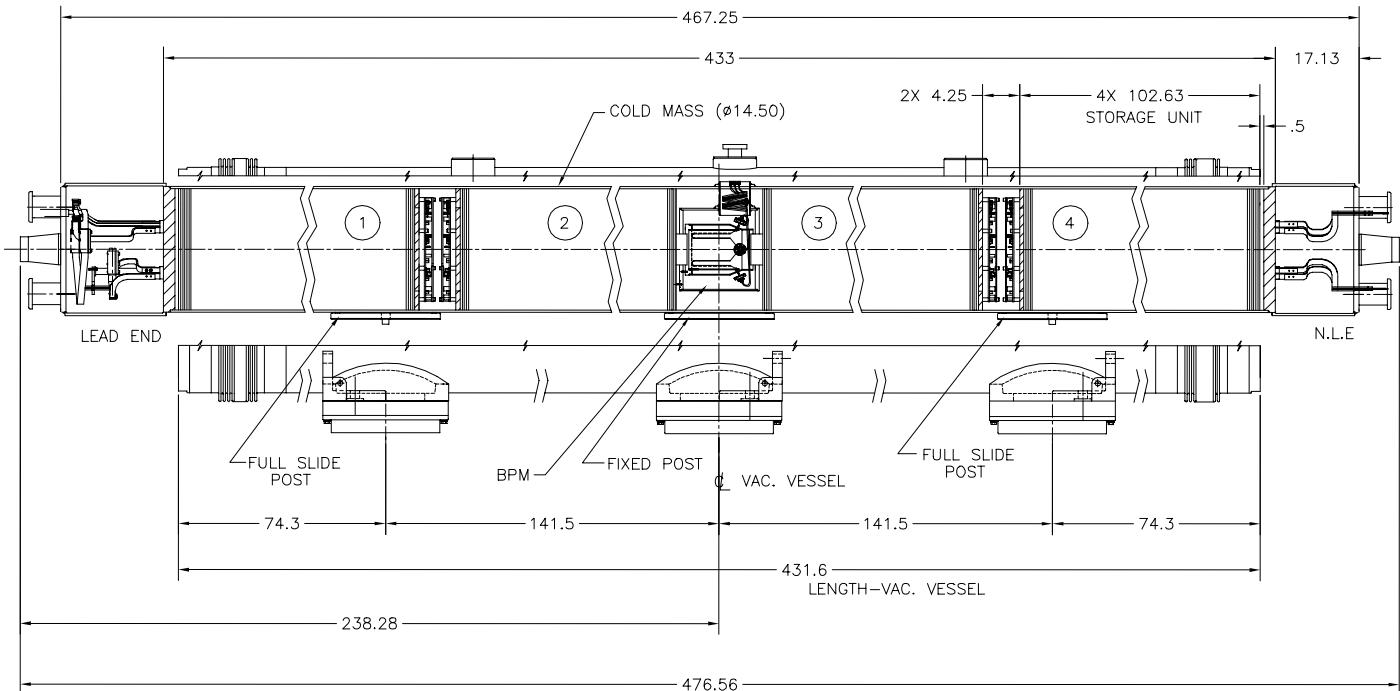
MJS, BNL







1. MODEL - 01, YELLOW RING WITH MODEL 01 COLD MASS.
2. MODEL - 02, BLUE RING WITH MODEL 01 COLD MASS.



HELICAL DIPOLE MAGNET  
DIMENSIONAL DRAWING  
"SNAKE"

Table 1: Magnets (“Storage Units”)

Magnet Name	Helicity	Field Orientation at Entrance/Exit	Quantity Required
HRCxxx	Right-Handed	Vertical	16
HRDxxx	Right-Handed	Horizontal	16
HRExxx	Left-Handed	Horizontal	16

Table 2: Complete Cryostat Assemblies

Unit Type	Field Orientation	Helicity Pattern(*)	Quantity Required	W-to-C transition(**)	Unit Name
Snake:	Vertical	RH,RH,RH,RH	4	(none)	(TBD)
Rotator: (Blue/R)	Horizontal	RH,LH,RH,LH	2	RHS of unit	(TBD)
Rotator: (Blue/L)	Horizontal	RH,LH,RH,LH	2	LHS of unit	(TBD)
Rotator: (Yellow/R)	Horizontal	LH,RH,LH,RH	2	RHS of unit	(TBD)
Rotator: (Yellow/L)	Horizontal	LH,RH,LH,RH	2	LHS of unit	(TBD)

\*The helicity pattern for the Snakes and Rotators are the pattern seen if standing at the inside wall of the tunnel, looking radially outward.

\*\*The Warm-to-Cold transition occurs either on the Right-hand side of the unit, or on the Left-hand side of the unit, as seen if standing at the inside wall of the tunnel, looking radially outward.

Parameter		Requirement	Tolerance (rms)
Design Central Field	$B_0$	4 Tesla	
Operating Margin		15 %	
Design Magnetic Length	$\frac{1}{B_0} \int  B  dL$	240 cm	
Magnet Slot Length		260.65 cm	
Total Cryostat Assembly Length		1186.815 cm	
Integrated Field Strength	$\int  B  d\ell$	9.6 Tesla-meter	0.05 T-m
Integrated Field Components	$\int B_x d\ell, \int B_y d\ell$	0 Gauss-m	500 Gauss-m
Quadrupole Coefficient of main dipole field	$b_1$	0	2.0
Sextupole Coefficient of main dipole field	$b_2$	2.0	2.0
Octupole Coefficient of main dipole field	$b_3$	0	2.0
Decapole Coefficient of main dipole field	$b_4$	2.0	1.0
Skew Quadrupole Coefficient of main dipole field	$a_1$	0	2.0
Transverse Alignment(*)	$\Delta x, \Delta y$	0 mm	0.5 mm
Longitudinal Alignment(*)	$\Delta z$	0 mm	1.0 cm
Rotational Alignment(*)	$\Delta \phi$	0 mrad	1.0 mrad

Table 6.3: General parameters and tolerances for an individual Snake or Rotator magnet. Magnet multipole coefficients are in units of  $10^{-4}$  at 3.1 cm reference radius. (\*)Alignment is with respect to neighboring quadrupoles.

Table 6: RHIC Spin Helical Dipole Magnets --- **Blue Ring**

Site Wide Name	Magnet Name	Type	Inside/ Outside	Sector	Location Number	Handedness	Field Orientation at End
<b>bo3-snk7</b>							
bo3-hlx7.4	HRCxxx	Sn	o	3	7	R	V
bo3-hlx7.3	HRCxxx	Sn	o	3	7	R	V
bo3-hlx7.2	HRCxxx	Sn	o	3	7	R	V
bo3-hlx7.1	HRCxxx	Sn	o	3	7	R	V
<b>bi5-rot3</b>							
bi5-hlx3.4	HRDxxx	Ro	i	5	3	R	H
bi5-hlx3.3	HRExxx	Ro	i	5	3	L	H
bi5-hlx3.2	HRDxxx	Ro	i	5	3	R	H
bi5-hlx3.1	HRExxx	Ro	i	5	3	L	H
<b>bo6-rot3</b>							
bo6-hlx3.1	HRDxxx	Ro	o	6	3	R	H
bo6-hlx3.2	HRExxx	Ro	o	6	3	L	H
bo6-hlx3.3	HRDxxx	Ro	o	6	3	R	H
bo6-hlx3.4	HRExxx	Ro	o	6	3	L	H
<b>bo7-rot3</b>							
bo7-hlx3.4	HRDxxx	Ro	o	7	3	R	H
bo7-hlx3.3	HRExxx	Ro	o	7	3	L	H
bo7-hlx3.2	HRDxxx	Ro	o	7	3	R	H
bo7-hlx3.1	HRExxx	Ro	o	7	3	L	H
<b>bi8-rot3</b>							
bi8-hlx3.1	HRDxxx	Ro	i	8	3	R	H
bi8-hlx3.2	HRExxx	Ro	i	8	3	L	H
bi8-hlx3.3	HRDxxx	Ro	i	8	3	R	H
bi8-hlx3.4	HRExxx	Ro	i	8	3	L	H
<b>bi9-snk7</b>							
bi9-hlx7.4	HRCxxx	Sn	i	9	7	R	V
bi9-hlx7.3	HRCxxx	Sn	i	9	7	R	V
bi9-hlx7.2	HRCxxx	Sn	i	9	7	R	V
bi9-hlx7.1	HRCxxx	Sn	i	9	7	R	V

Table 7: RHIC Spin Helical Dipole Magnets --- **Yellow Ring**

Site Wide Name	Magnet Name	Type	Inside/ Outside	Sector	Location Number	Handedness	Field Orientation at End
<b>yi3-snk7</b>							
yi3-hlx7.4	HRCxxx	Sn	i	3	7	R	V
yi3-hlx7.3	HRCxxx	Sn	i	3	7	R	V
yi3-hlx7.2	HRCxxx	Sn	i	3	7	R	V
yi3-hlx7.1	HRCxxx	Sn	i	3	7	R	V
<b>yo5-rot3</b>							
yo5-hlx3.4	HRExxx	Ro	o	5	3	L	H
yo5-hlx3.3	HRDxxx	Ro	o	5	3	R	H
yo5-hlx3.2	HRExxx	Ro	o	5	3	L	H
yo5-hlx3.1	HRDxxx	Ro	o	5	3	R	H
<b>yi6-rot3</b>							
yi6-hlx3.1	HRExxx	Ro	i	6	3	L	H
yi6-hlx3.2	HRDxxx	Ro	i	6	3	R	H
yi6-hlx3.3	HRExxx	Ro	i	6	3	L	H
yi6-hlx3.4	HRDxxx	Ro	i	6	3	R	H
<b>yi7-rot3</b>							
yi7-hlx3.4	HRExxx	Ro	i	7	3	L	H
yi7-hlx3.3	HRDxxx	Ro	i	7	3	R	H
yi7-hlx3.2	HRExxx	Ro	i	7	3	L	H
yi7-hlx3.1	HRDxxx	Ro	i	7	3	R	H
<b>yo8-rot3</b>							
yo8-hlx3.1	HRExxx	Ro	o	8	3	L	H
yo8-hlx3.2	HRDxxx	Ro	o	8	3	R	H
yo8-hlx3.3	HRExxx	Ro	o	8	3	L	H
yo8-hlx3.4	HRDxxx	Ro	o	8	3	R	H
<b>yo9-snk7</b>							
yo9-hlx7.4	HRCxxx	Sn	o	9	7	R	V
yo9-hlx7.3	HRCxxx	Sn	o	9	7	R	V
yo9-hlx7.2	HRCxxx	Sn	o	9	7	R	V
yo9-hlx7.1	HRCxxx	Sn	o	9	7	R	V

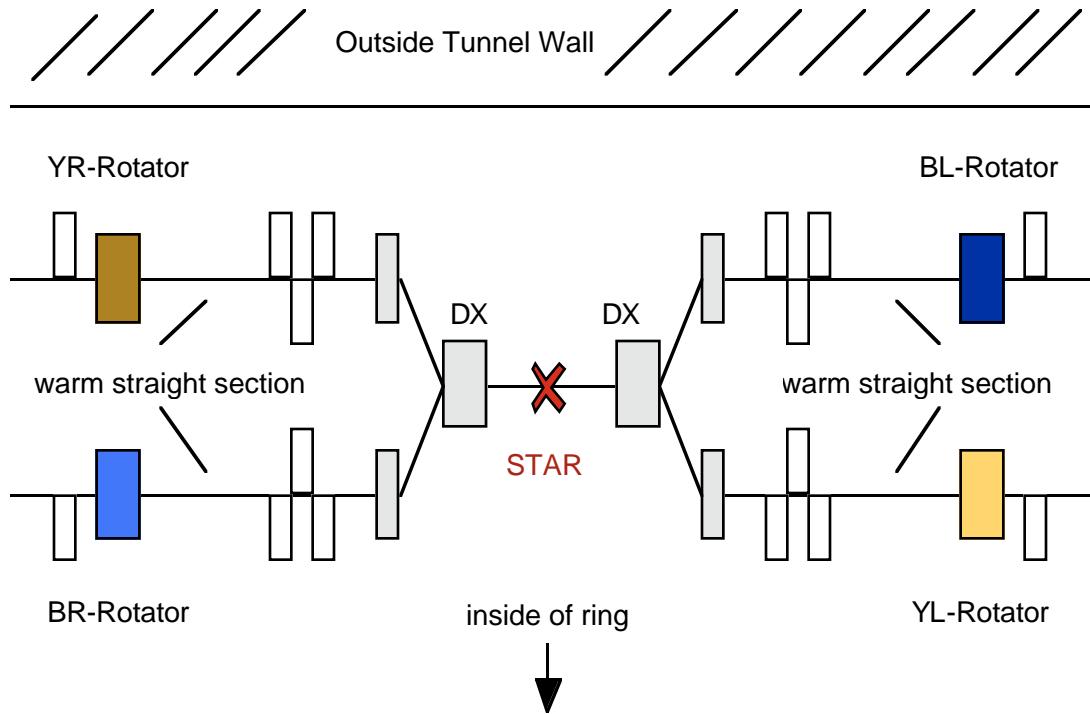


Figure 2: Schematic layout of Rotators in STAR region

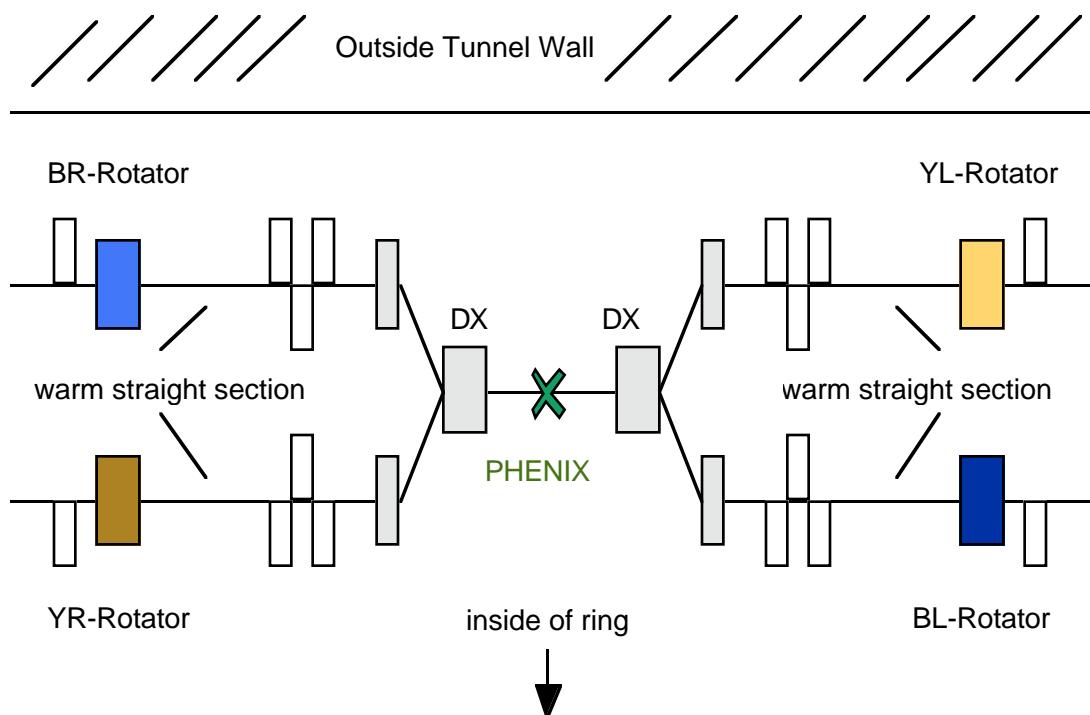


Figure 3: Schematic layout of Rotators in PHENIX region

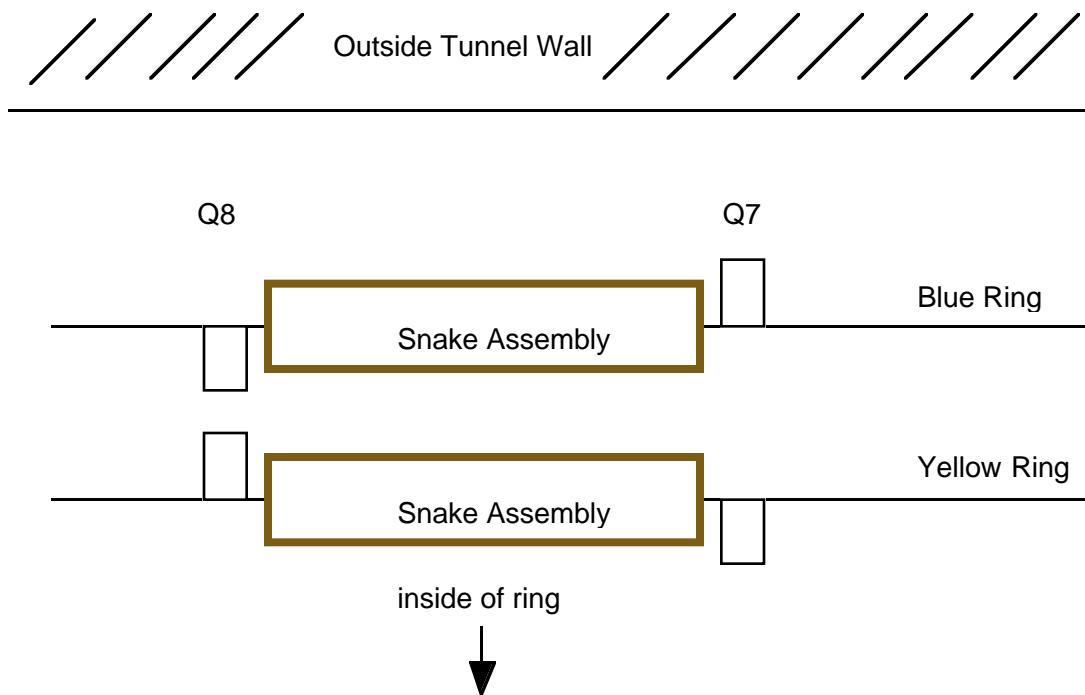


Figure 4: Schematic layout of Snakes in 3 o'clock region

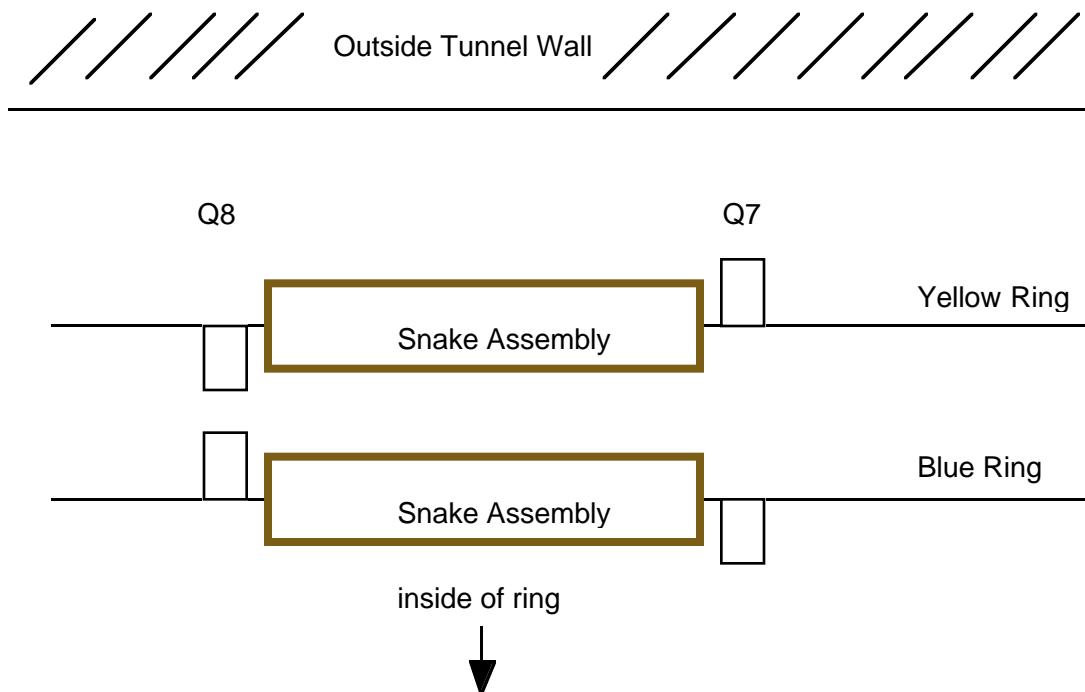


Figure 5: Schematic layout of Snakes in 9 o'clock region